What are Eigenvalues and Eigenvectors?

Eigenvectors and eigenvalues live in the heart of the data science field. This article will aim to explain what eigenvectors and eigenvalues are, how they are calculated and how we can use them. It’s a must-know topic for anyone who wants to understand machine learning in-depth.

Eigenvalues and eigenvectors form the basics of computing and mathematics. They are heavily used by scientists.

# 1. Eigenvectors and Eigenvalues Introduction

**Before we take a deep dive into calculating eigenvectors and eigenvalues, let’s understand what they really are.**

Let’s consider that we want to build mathematical models (equations) where the input data is gathered from a large number of sources. As an instance, let’s assume that we want to forecast a complex financial variable, such as the behavior of interest rates over time. Let’s refer to interest rates as y.

The first step might involve finding the variables that y is dependent on. Let’s refer to these variables as **x(i)**

We will start our research by gathering data for variables that **y** is dependent on. Some of the data might be in textual format. The task would be to convert the non-numerical data into numerical data. As an instance, we often use one-hot encoding to transform values in textual features to separate numerical columns. If our input data is in images format then we would have to somehow convert the image into numerical matrices.

The second step would be to join the data into a tabular format where each column of the table is computed by 1 or more features. This will result in a large sparse matrix (table). At times, it can increase our dimension space to 100+ columns.

Now let’s understand this!

It introduces its own sets of problems such as the large sparse matrix can end up taking a significant amount of space on a disk. Plus, it becomes extremely time-consuming for the model to train itself on the data. Furthermore, it is difficult to understand and visualize data with more than 3 dimensions, let alone a dataset of over 100+ dimensions. Hence, it would be ideal to somehow compress/transform this data into a smaller dataset.

There is a solution. We can utilise Eigenvalues and Eigenvectors to reduce the dimension space. To elaborate, one of the key methodologies to improve efficiency in computationally intensive tasks is to reduce the dimensions after ensuring most of the key information is maintained.

Eigenvalues and Eigenvectors are the key tools to use in those scenarios

## **1.1 What Is An Eigenvector?**

I would like to explain this concept in a way that we can easily understand it.

For the sake of simplicity, let’s consider that we live in a two-dimensional world.

* Alex’s house is located at coordinates [10,10] (x=10 and y =10). Let’s refer to it as vector A.
* Furthermore, his friend Bob lives in a house with coordinates [20,20] (x=20 and y=20). I will refer to it as vector B.

If Alex wants to meet Bob at his place then Alex would have to travel +10 points on the x-axis and +10 points on the y-axis. This movement and direction can be represented as a two-dimensional vector [10,10]. Let’s refer to it as vector C.

We can see that vector A to B are related because vector B can be achieved by scaling (multiplying) the vector A by 2. This is because 2 x [10,10] = [20,20]. This is the address of Bob. Vector C also represents the movement for A to reach B.

The key to note is that a vector can contain the magnitude and direction of a movement. So far so good!

We learned from the introduction above that large set of data can be represented as a matrix and we need to somehow compress the columns of the sparse matrix to speed up our calculations. Plus if we multiply a matrix by a vector then we achieve a new vector. The multiplication of a matrix by a vector is known as transformation matrices.

We can transform and change matrices into new vectors by multiplying a matrix with a vector. The multiplication of the matrix by a vector computes a new vector. This is the transformed vector. Hold that thought for now!

The new vector can be considered to be in two forms:

1. Sometimes, the new transformed vector is just a scaled form of the original vector. This means that the new vector can be re-calculated by simply multiplying a scalar (number) to the original vector; just as in the example of vector A and B above.
2. And other times, the transformed vector has no direct scalar relationship with the original vector which we used to multiply to the matrix.

If the new transformed vector is just a scaled form of the original vector then the original vector is known to be an eigenvector of the original matrix. Vectors that have this characteristic are special vectors and they are known as eigenvectors. Eigenvectors can be used to represent a large dimensional matrix.

Therefore, if our input is a large sparse matrix M then we can find a vector o that can replace the matrix M. The criteria is that the product of matrix M and vector o should be the product of vector o and a scalar n:

M \* o = n\* o

This means that a matrix M and a vector o can be replaced by a scalar n and a vector o.

**In this instance, o is the eigenvector and n is the eigenvalue and our target is to find o and n.**

Therefore an eigenvector is a vector that does not change when a transformation is applied to it, except that it becomes a scaled version of the original vector.

Eigenvectors can help us calculating an approximation of a large matrix as a smaller vector. There are many other uses which I will explain later on in the article.

Eigenvectors are used to make linear transformation understandable. Think of eigenvectors as stretching/compressing an X-Y line chart without changing their direction.

## **1.2 What is an Eigenvalue?**

**Eigenvalue**— The scalar that is used to transform (stretch) an Eigenvector.

Let’s understand where eigenvalues and eigenvectors are used

# 2. Where are Eigenvectors and Eigenvalues used?

There are multiple uses of eigenvalues and eigenvectors:

1. Eigenvalues and Eigenvectors have their importance in linear differential equations where you want to find a rate of change or when you want to maintain relationships between two variables.

Think of eigenvalues and eigenvectors as providing summary of a large matrix

2. We can represent a large set of information in a matrix. Performing computations on a large matrix is a very slow process. To elaborate, one of the key methodologies to improve efficiency in computationally intensive tasks is to reduce the dimensions after ensuring most of the key information is maintained. Hence, one eigenvalue and eigenvector are used to capture key information that is stored in a large matrix. This technique can also be used to improve the performance of data churning components.

3. Component analysis is one of the key strategies that is utilised to reduce dimension space without losing valuable information. **The core of component analysis (PCA) is built on the concept of eigenvalues and eigenvectors.**The concept revolves around computing eigenvectors and eigenvalues of the covariance matrix of the features.

4. Additionally, eigenvectors and eigenvalues are used in facial recognition techniques such as EigenFaces.

5. They are used to reduce dimension space. The technique of Eigenvectors and Eigenvalues are used to compress the data. As mentioned above, many algorithms such as PCA rely on eigenvalues and eigenvectors to reduce the dimensions.

6. Eigenvalues are also used in regularisation and they can be used to prevent overfitting.

Eigenvectors and eigenvalues are used to reduce noise in data. They can help us improve efficiency in computationally intensive tasks. They also eliminate features that have a strong correlation between them and also help in reducing over-fitting.